4.5 Negative Exponents and Reciprocals

LESSON FOCUS	Relate negative exponents to reciprocals.

Copy & Complete the following:

Ex: $\frac{2^4}{2^2} = 2^2$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2} = 2^2 = 4$	$\frac{16}{4} = 4$
$\frac{2^4}{2^3} = 2^1$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2} = 2$	$\frac{16}{8} = 2$
$\frac{2^4}{2^4} = 2^0$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2\cdot2} = 1$	$\frac{16}{16} = 1$
$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2\cdot2\cdot2} = \frac{1}{2}$	$\frac{16}{32} = \frac{1}{2}$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$\frac{2^4}{2^6} = 2^{-2} = \frac{1}{2^2}$

$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

$\frac{3}{4}$ $\frac{4}{3}$

Copy & Complete the following:

Ex: $\frac{2^4}{2^2} = 2^2$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^2$	$\frac{16}{4} = 4$
$\frac{2^4}{2^3} = 2^1$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^1$	$\frac{16}{8} = 2$
$\frac{2^4}{2^4} = 2^0$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = 2^0$	$\frac{16}{16} = 1$
$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2^{-1}$	$\frac{16}{32} = \frac{1}{2}$

$(-2)^{-3} = \frac{1}{(-2)^3}$

$\frac{1}{(-3)^{-4}} = (-3)^4$

$\frac{1}{4^{-3}} = 4^3$

$$9^{-\frac{3}{2}} = \frac{1}{9^{3/2}} = \frac{1}{\left(\sqrt{9}\right)^3}$$

$$= \frac{1}{3^3} = \frac{1}{27}$$

Ex:

$$27^{-\frac{2}{3}} = \frac{1}{27^{2/3}} = \frac{1}{\left(\sqrt[3]{27}\right)^2} = \frac{1}{(3)^2} = \frac{1}{9}$$

Negative Exponent Law

Fractional Exponents

$$100^{-\frac{3}{2}} = \frac{1}{100^{3/2}} = \frac{1}{\left(\sqrt{100}\right)^3} = \frac{1}{10^3}$$

$$= \frac{1}{1000}$$

$$= 0.001$$

TRY THIS

Work with a partner.

You will need grid paper and scissors.

A. Cut out a 16 by 16 grid. Determine the area of the grid in square units and as a power of 2. Record your results in a table like this:

Cut	Area (units^2)	Area as a Power of 2
Start	256	
1		
2		
3		

B. Cut the grid in half and discard one piece.
In the table, record the area of the remaining piece in square units and as a power of 2.

C. Repeat Step B until the paper cannot be cut further.

Activity

Two numbers with a product of 1 are reciprocals.

Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals.

Similarly, $\frac{2}{3} \cdot \frac{3}{2} = 1$, so the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are also reciprocals.

We define powers with negative exponents so that previously developed properties such as $a^m \cdot a^n = a^{m+n}$ and $a^0 = 1$ still apply.

Apply these properties.

$5^{-2} \cdot 5^2 = 5^{-2+2}$

$= 5^0$

$= 1$

Since the product of 5^{-2} and 5^2 is 1, 5^{-2} and 5^2 are reciprocals.

So, $5^{-2} =$? and $\frac{1}{5^{-2}} =$?

That is, $5^{-2} =$?

This suggests the following definition for powers with negative exponents.

NEGATIVE EXPONENTS

copy

x^{-n} is defined to be the reciprocal of x^n

That is $x^{-n} = \frac{1}{x^n}$, $(x \neq 0)$

~~Exponent Law #5~~

NEGATIVE EXPONENTS

$3^{-1} = \frac{1}{3}$

Remember that a negative exponent does not mean a negative number but the *reciprocal number.*

copy

$4^{-3} = \frac{1}{4^3}$

WHY IS THIS TRUE?

- WATCH THIS PATTERN
- $10^3 = 1000$
- $10^2 = 100$
- $10^1 = 10$
- $10^0 = 1$
- $10^{-1} = 0.1$ or $\frac{1}{10}$
- $10^{-2} = 0.01$ or $\frac{1}{100}$

Zero Exponent –

Negative Exponent –

EXAMPLE

copy

Long Version	Short Version
$\frac{4^{-5}}{4^{-3}} = 4^{-5} \div 4^{-3}$ (-5+(+3))	$\frac{4^{-5}}{4^{-3}} = \frac{4^3}{4^5} = \frac{1}{4^2}$
$= \frac{1}{4^5} \div \frac{1}{4^3}$	Flip Reciproc make positive powers
$= \frac{1}{4^5} \times \frac{4^3}{1}$	Subtract powers and place where there are more.
$= \frac{4^3}{4^5} = 4^{-2} = \boxed{\frac{1}{4^2}}$	

copy

Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n.

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n, x \neq 0$

4.5 Negative Exponents and Reciprocals

Wednesday, October 5

- Unit 1 test postponed until Wednesday, October 12
- Today:
- Review negative exponents
- Review exponent laws learned last year
- Practice using exponents laws and combination of laws
- Class work/Homework

Example 1 Evaluating Powers with Negative Integer Exponents

Evaluate each power.

Please copy question and answer (examples for your notes).

a) 3^{-2}

$= \frac{1}{3^2} = \frac{1}{9}$

b) $\left(-\frac{3}{4}\right)^{-3}$

$= \left(-\frac{4}{3}\right)^{3}$

$= \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right) =$

$\frac{-64}{27} = -2\frac{10}{27}$

c) 0.3^{-4}

CHECK YOUR UNDERSTANDING

SOLUTION

Example 1 Evaluating Powers with Negative Integer Exponents

Evaluate each power.

a) 3^{-2} b) $\left(-\frac{3}{4}\right)^{-3}$ c) 0.3^{-4}

SOLUTION

a) $3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

b) $\left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^{3}$

$= -\frac{64}{27}$

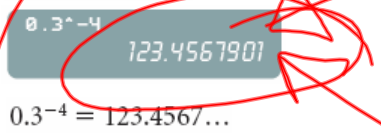
(Solution continues.)

4.5 Negative Exponents and Reciprocals

Example 1 Evaluating Powers with Negative Integer Exponents

c) 0.3^{-4}

Use a calculator.



$0.3^{-4} = 123.4567\ldots$

$\frac{1}{0.3^4} = \frac{1}{0.0081} = \frac{10000}{81} = 123\frac{37}{81}$

CHECK YOUR UNDERSTANDING

copy

$8^{-\frac{2}{3}}$

$-\frac{2}{3}$ is the product: $(-1)\left(\frac{1}{3}\right)(2)$, and order does not matter

we can apply the three operations

cube root, reciprocal, and square in any order.

Example 2 Evaluating Powers with Negative Rational Exponents

Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}}$

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

SOLUTION

CHECK YOUR UNDERSTANDING

Example 2 Evaluating Powers with Negative Rational Exponents

Evaluate each power without using a calculator.

a) $8^{-\frac{2}{3}}$ b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

SOLUTION

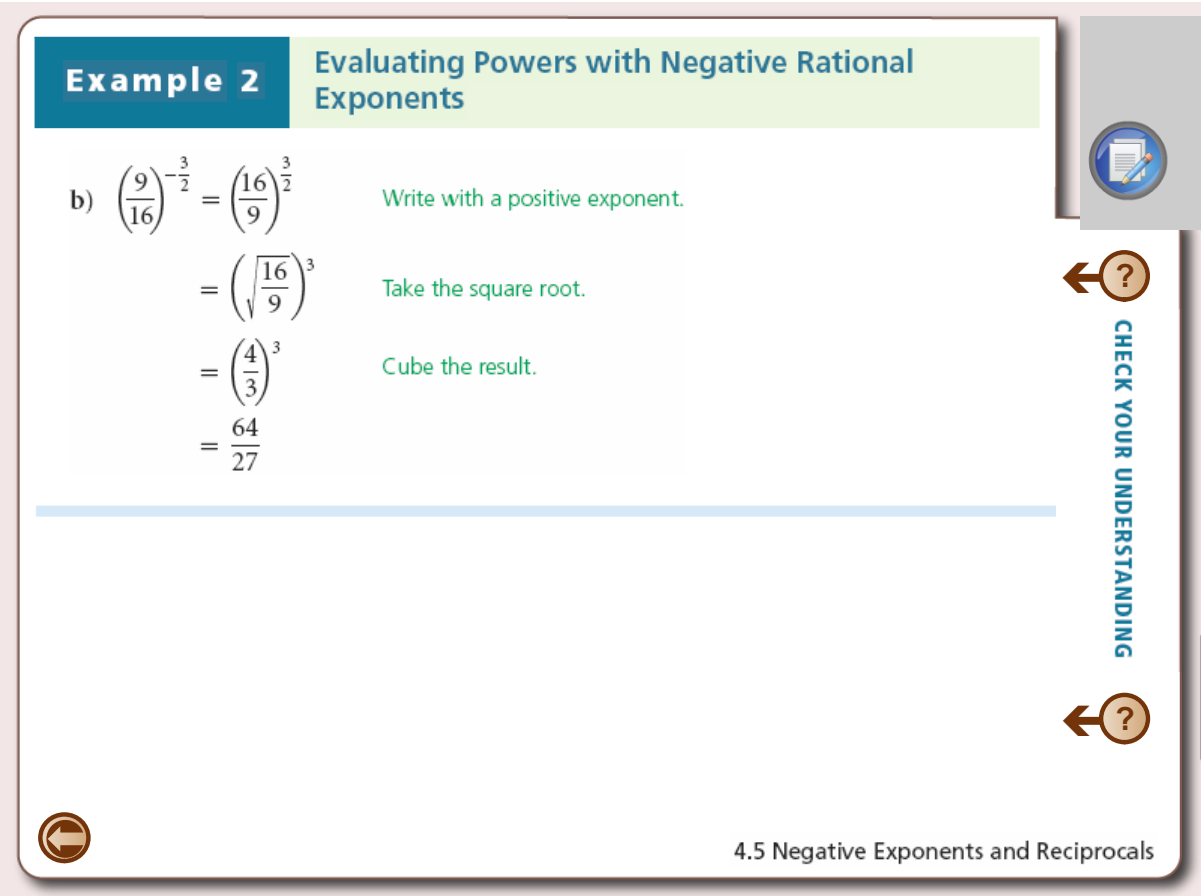
a) $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}}$ Write with a positive exponent.

$= \frac{1}{(\sqrt[3]{8})^2}$ Take the cube root.

$= \frac{1}{2^2}$ Square the result.

$= \frac{1}{4}$

(Solution continues.)



Thursday, October 6

- Unit 1 test postponed until Wednesday, October 12
- Today:
- Check and go over questions on negative exponents
- Review exponent laws learned last year
- Practice using exponents laws and combination of laws
- Class work/Homework

Review assignment due Wednesday, Oct.12

Classwork/Homework

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hardest

$2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024}$

3. Copy then complete each equation.

a) $\frac{1}{5^4} = 5^{\square}$ -4 b) $\left(-\frac{1}{2}\right)^{-3} = (-2)^{\square}$ 3

c) $\frac{1}{3^{\square}} = 3^2$ -2 d) $\frac{1}{4^{-2}} = 4^{\square}$ 2

5. Given that $2^{10} = 1024$, what is 2^{-10}? $2^{-10} = \frac{1}{2^{10}} = \frac{1}{1024}$

6. Write each power with a positive exponent.

a) 2^{-3} $\frac{1}{2^3}$ b) 3^{-5} $\frac{1}{3^5}$ c) $(-7)^{-2}$ $\frac{1}{(-7)^2}$

7. Write each power with a positive exponent.

a) $\left(\frac{1}{2}\right)^{-2} = 2^2$ b) $\left(\frac{2}{3}\right)^{-3}$ $\left(\frac{3}{2}\right)^3$ c) $\left(-\frac{6}{5}\right)^{-4}$ $\left(-\frac{5}{6}\right)^4$

8. Evaluate each power without using a calculator.

a) 3^{-2} b) 2^{-4} c) $(-2)^{-5}$

d) $\left(\frac{1}{3}\right)^{-3}$ e) $\left(-\frac{2}{3}\right)^{-2}$ f) $\frac{1}{5^{-3}}$

a) $\frac{1}{3^2} = \frac{1}{9}$ b) $\frac{1}{2^4} = \frac{1}{16}$

c) $\frac{1}{(-2)^5} = \frac{1}{-32}$

d) $3^3 = 27$

e) $\left(-\frac{3}{2}\right)^2 = \left(-\frac{3}{2}\right)\left(-\frac{3}{2}\right) = \frac{9}{4}$

f) $5^3 = 125$

9. Evaluate each power without using a calculator.

a) $4^{-\frac{1}{2}}$ **b)** $0.09^{-\frac{1}{2}}$

c) $27^{-\frac{1}{3}}$ **d)** $(-64)^{-\frac{1}{3}}$

e) $(-0.027)^{-\frac{2}{3}}$ **f)** $32^{-\frac{2}{5}}$

g) $9^{-\frac{3}{2}}$ **h)** $0.04^{-\frac{3}{2}}$

a) $\frac{1}{4^{1/2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$

c) $\frac{1}{27^{1/3}} = \frac{1}{\sqrt[3]{27}} = \frac{1}{3}$

f) $\frac{1}{32^{2/5}} = \frac{1}{(\sqrt[5]{32})^2} = \frac{1}{2^2} = \frac{1}{4}$

10. Use a power with a negative exponent to write an equivalent form for each number.

a) $\frac{1}{9}$ **b)** $\frac{1}{5}$ **c)** 4 **d)** -3

a) $= \frac{1}{3^3} = \frac{1}{27}$

g) $\frac{1}{9^{3/2}} = \frac{1}{(\sqrt{9})^3} = \frac{1}{3^3} = \frac{1}{27}$

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13. Evaluate each power without using a calculator.

a) $27^{-\frac{4}{3}}$ **b)** $16^{-1.5}$ **c)** $32^{-0.4}$

d) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$ **e)** $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ **f)** $\left(\frac{9}{4}\right)^{-\frac{5}{2}}$

$\left(-\frac{27}{8}\right)^{\frac{2}{3}}$

$\left(\sqrt[3]{\left(-\frac{27}{8}\right)}\right)^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

13. Evaluate each power without using a calculator.

a) $27^{-\frac{4}{3}}$ **b)** $16^{-1.5}$ **c)** $32^{-0.4}$

d) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$ **e)** $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ **f)** $\left(\frac{9}{4}\right)^{-\frac{5}{2}}$

a) $\frac{1}{27^{4/3}} = \frac{1}{\left(\sqrt[3]{27}\right)^4} = \frac{1}{3^4} = \frac{1}{81}$

d) $\left(-\frac{27}{8}\right)^{\frac{2}{3}} = \left(\sqrt[3]{\left(-\frac{27}{8}\right)}\right)^2 = \left(-\frac{3}{2}\right)^2$

$= \frac{9}{4} = 2\frac{1}{4}$

a) $27^{-\frac{4}{3}}$ b) $16^{-1.5}$ c) $32^{-0.4}$

d) $\left(-\frac{8}{27}\right)^{-\frac{2}{3}}$ e) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ f) $\left(\frac{9}{4}\right)^{-\frac{5}{2}}$

b) $16^{-3/2} = \frac{1}{16^{3/2}} = \frac{1}{\left(\sqrt{16}\right)^3} = \frac{1}{4^3} = \frac{1}{64}$

a) $\frac{1}{27^{4/3}} = \frac{1}{\left(\sqrt[3]{27}\right)^4} = \frac{1}{3^4} = \frac{1}{81}$

f) $\left(\frac{4}{9}\right)^{5/2} = \left(\sqrt{\frac{4}{9}}\right)^5 = \left(\frac{2}{3}\right)^5 = \frac{32}{243}$

We are now done all new information for this Unit.

However, we are now going to practice/review exponent laws learned last year with these 2 new ones.